purities present in the interface region. Thus, it would be expected that the 15- to 25-μsec test time reported in Ref. 2 is greater than would be found by other means of indicating test time.

Figure 1 shows results of measurements of test time for the 1.5-in.-diam shock tube made with photomultipliers and electron collector probes. The photomultiplier was focused on the center of the tube through a series of collimated slits so that a spatial resolution of approximately 1 mm was obtained. The electron collector probe was a simple wire extended into the flow and biased with a positive voltage (+6 v above shock tube ground) so that electrons are attracted to the probe. Some uncertainty in test time exists in both of these measuring methods in that neither radiation nor ionization occurs directly at the shock front. The uncertainty due to the finite adjustment time should be of the order of 0.5 μsec at the higher shock Mach numbers (based on the extrapolation of ionization time measurements). For the photomultiplier measurements, shown in Fig. 1, below a shock Mach number of 15, radiation from impurities in the driven gas was sufficient to indicate the shock front. A nonequilibrium overshoot in radiation,[4] which lasts for an average of 2 μsec, is present in the photomultiplier traces. Thus, for $M_s \approx 24$ and an initial pressure of 250 μ Hg, equilibrium test times from 6 to 4 μsec are indicated. No change in the measured heat transfer rates of Ref. 2 was found when the data were re-evaluated using the forementioned results.

Hoshizaki's question was based on theoretical test times calculated from the analysis of Roshko.[5] Roshko's analysis has been improved by Camm and Rose,[4] and their results have been used to predict the shaded curves, shown for the different initial pressures, on Fig. 1. The upper limit of each curve is that predicted directly from theoretical considerations. Experimental measurements of shock tube test times at low shock Mach numbers reported previously[6] have been re-analyzed in the light of Camm and Rose's improved theory, and the data are found to follow closely the trends of the theory. However, it was found that the data are systematically lower than the predicted values. The lower limit of the shaded curves was determined from a faired curve through the data. As can be seen, the present test time measurements at $P_1 = 1$ and 0.5 mm Hg are in good agreement with the low shock Mach number measurements.[6] A boundary layer thickness parameter β enters directly into the determination of the curves shown on Fig. 1. The parameter β is computed from laminar boundary layer solutions obtained by Mirels.[7] The solutions of Mirels are for initial pressures of 0.76 and 7.6 mm Hg and a range of shock Mach numbers between 4 and 14. Outside this range of conditions an extrapolation formula is given. For the low initial pressure conditions ($p_1 \leq 0.25$ mm Hg), it appears that the value of β is lower than would be predicted by extrapolation. This may explain the low values of test time computed by Hoshizaki (2 to 3 μsec).

The values of test time shown in Fig. 1 are at greater distances down the shock tube than the original data. The data below a Mach number of 15 were at a station 22.6 ft from the diaphragm, and the high Mach number data were taken at a station 23.7 ft from the diaphragm. The data of Ref. 2 were taken at a station 16 ft from the diaphragm. A reduction of 5% in test time at $p_1 = 0.25$ mm Hg would be predicted from the theoretical calculations if the length of the shock tube were reduced from 23.7 to 16 ft. (The theoretical curves of Fig. 1 are for a length of 23.7 ft.) Thus, the effect of the shorter length is negligible for this case. For greater initial pressures the reduction in test time increases.

According to the theoretical analysis of low-pressure shock tube test time, the time should increase as the square of the tube diameter. The low Mach number data of Ref. 6 were taken in a 3-in.-diam tube. The agreement of the $p_1 = 1$ and 0.5 mm Hg data with the predicted curves indicates that the scaling is reasonable. On the other hand, the test

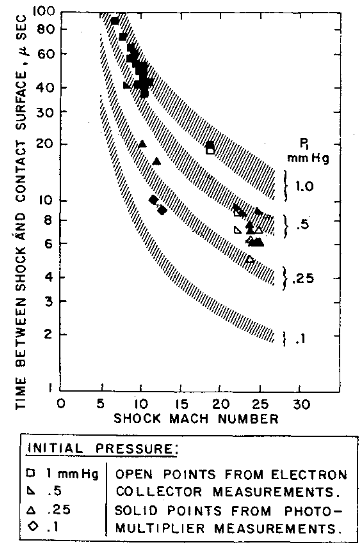

Fig. 1 Test time measurements in 1.5-in.-diam shock tube compared with predicted test time

times obtained by Camm and Rose[4] for a 6-in.-diam shock tube fall far below theoretical values. (Similar results have been obtained for a 6.5-in.-diam shock tube at the Avco Research and Advanced Development Division.) The 6-in.-diam tube had a 1.328-in.-diam driver and expands out (after the diaphragm) to the 6-in. diam. Apparently the expansion process greatly reduces the test time. The larger tube gives longer test times than the 1.5-in.-diam tube, but the gain is far less than expected from theoretical considerations.

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Comment on the Soap-Film Paradox

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In a previous paper,[1] Gellatly discussed a seeming contradiction in the calculation of the equilibrium configuration of a soap film hanging from a horizontal ring. The paradox may be seen in an even simpler configuration, namely, a soap film hanging in a vertical, plane wire frame. Gellatly

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[1] Gellatly, R. A., "A note on a soap-film paradox," J. Aerospace Sci. **29**, 1487 (1962).

uses two assumptions: 1) uniform surface tension T, and 2) static equilibrium. With these, one concludes that the net force on each element of the vertical film is its weight. Therefore, the film should fall with the acceleration of gravity. Actually, experimental observation shows relatively small downward motions and accelerations.

With the two assumptions mentioned, the film *cannot* be in static equilibrium, either in the case adduced by the author or in the case of the vertical film just cited. The self-contradictory result of the analysis is a signal that the assumptions are not valid. The physical model employed for the analysis should be improved in three respects:

1) There is downward drainage of liquid in the film, opposed by viscous forces. By reason of this motion, one may not suppose static equilibrium to prevail, and viscosity must be taken into account.

2) A further result of the motion is that the chemical concentration in the surface layer of molecules changes. The gradients in concentration produce gradients in surface tension T. Such variations in surface tension, it is well known, account for the "mechanical strength" of foams, i.e., their resistance to collapsing under their own weight.

3) In formulating a theory of the problem, with motion included, one also should take account of the "surface viscosity," i.e., the resistance to deformation of areas in the surface.

Comment on the Soap-Film Paradox

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In a recent note,[1] Gellatly discussed a soap-film paradox. According to the analysis given there, the two equations of equilibrium for forces acting on a film element in the horizontal and the vertical directions are, respectively,

$$\sin\phi[\tan\phi + r(d\phi/dr)] = 0 \tag{1}$$

$$\cos\phi[\tan\phi + r(d\phi/dr) - (Wr/2T\cos^2\phi)] = 0 \tag{2}$$

The equations are essentially Eqs. (2) and (6) in Gellatly's note but are presented here in a slightly different form for the convenience of the following discussion. The notation is that of Gellatly.

The paradox is essentially that there is no meaningful solution for these two equations except

$$\phi = 0 \qquad W/2T = 0 \tag{3}$$

which represent a flat film under zero vertical load W or under an infinitely large tension $2T$.

First it should be known whether the derivation leading to Eqs. (1) and (2) can be reconciled with the classical linear formulation of the membrane problem. If yes, in what sense? In the classical formulation, ϕ is assumed to be small, and all small quantities of second order are neglected. Equation (2) then leads to the well-known Poisson equation, and Eq. (1) is *ignored* because every term in that equation is a small quantity of second order. In other words, in the classical formulation the two equations of equilibrium are both satisfied up to and including the first power of ϕ.

For problems where ϕ may not be small, one needs to consider Eqs. (1) and (2) in their entirety. Then one finds that they are incompatible except for the trivial solution given by Eq. (3). This situation implies that the analysis is based upon an incorrect model, although the model is a "permissible" one for films with small deflections. It is worth noting here that, if the loading (denoted now by p) on the surface is in the direction normal to the surface, then the two equations of equilibrium become

$$\sin\phi\left(\tan\phi + r\frac{d\phi}{dr} - \frac{pr}{2T\cos\phi}\right) = 0 \tag{4}$$

$$\cos\phi\left(\tan\phi + r\frac{d\phi}{dr} - \frac{pr}{2T\cos\phi}\right) = 0 \tag{5}$$

and the paradox does not appear.

In relation to the paradox, a simpler and analogous problem of a rope hanging from two pegs will be mentioned. There it is quite obvious that if the tension in the rope is assumed to be *constant* the equilibrium condition in the horizontal direction for forces acting on any rope element never can be satisfied except when the rope is completely in a horizontal position. The corresponding equation of equilibrium for forces in the vertical direction then will demand the vertical load to be zero or the tension to be infinitely large. Hence, in general, one must admit variation of the tension along the rope. On the other hand, if the slope of the rope is everywhere small, then one can show that the rate of change of the tension along the rope is a small quantity of second order, and, therefore, the model of a rope under constant tension is a permissible one within the framework of a linear theory. The corresponding equations of equilibrium are satisfied up to the first order.

It is apparent that for the problem discussed by Gellatly the model of a film under constant tension $2T$ is not an adequate one. Many different models can be constructed in order to remove the paradox. One version will be presented which seems to be the most satisfactory. Consider a more realistic model of the film as consisting of three parts: an upper and a lower massless membrane and a fluid space in between. Each membrane is under a constant tension T. The upper membrane is exposed to the ambient pressure on the top surface and to a fluid pressure p_u on the bottom surface. If p_u is measured with respect to the ambient pressure (i.e., gage), then the governing equation for the geometry of the upper membrane is

$$r\frac{d\phi_u}{dr} + \tan\phi_u = -\frac{p_u r}{T\cos\phi_u} \tag{6}$$

The lower membrane is exposed to a fluid pressure p_b (gage) on the top surface and the ambient pressure (zero gage) on the bottom surface. Hence, the governing equation is

$$r\frac{d\phi_b}{dr} + \tan\phi_b = \frac{p_b r}{T\cos\phi_b} \tag{7}$$

If one denotes by w_u and w_b the elevations of the upper and lower surfaces of the film, then the pressures p_u and p_b may be expressed as

$$p_u = p_0 - \mu w_u \qquad p_b = p_0 - \mu w_b \tag{8}$$

where p_0 is a reference pressure as yet unknown and μ is the specific weight of the fluid. Substituting Eqs. (8) into Eqs. (6) and (7), one has

$$r\frac{d\phi_u}{dr} + \tan\phi_u = \frac{r}{T\cos\phi_u}(\mu w_u - p_0) \tag{9}$$

$$r\frac{d\phi_b}{dr} + \tan\phi_b = \frac{r}{T\cos\phi_b}(p_0 - \mu w_b) \tag{10}$$

Furthermore, one has also

$$dw_u/dr = \tan\phi_u \tag{11}$$

$$dw_b/dr = \tan\phi_b \tag{12}$$

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[1] Gellatly, R. A., "A note on a soap-film paradox," J. Aerospace Sci. 29, 1487 (1962).